Reservoir Simulator Practical

Course Notes 2012

Philipp Lang
IZR Room 403
Tel 3004
philipp.lang@unileoben.ac.at

for further information please refer to the accompanying document ‘Info Sheet & Course Logistics’
Module III: Three Phase Flow

The term ‘black oil’ in reservoir engineering refers to systems which contain all three phases encountered in hydrocarbon reservoirs – gas, oil, water – accounting for pressure-temperature dependent transition between oil and gas phase. In reservoir simulation, these systems are subject to significant non-linearity due to the mutual dependence of flow properties on saturation and saturation on pressure and temperature.

NB: These lecture notes are supposed to be read ‘in one piece’ as it also serves as a walk through for the lab exercises. The material presented will help you understand the concepts covered in class. It does not serve as a substitute for attending the course (considered as auxiliary only).

Black Oil Model ............................................................................................................................................... 1
Pressure and Fluid Properties....................................................................................................................... 1
The Formulation ............................................................................................................................................... 3
Three Phase Relative Permeabilities ............................................................................................................ 5
Finite-Difference Grids and Initialization ...................................................................................................... 7
The Grid ............................................................................................................................................................ 7
Initialization ....................................................................................................................................................... 8
Non-Linearity.................................................................................................................................................... 9
Related BSc Examination Concepts ............................................................................................................. 11
References....................................................................................................................................................... 12
Black Oil Model

Saturated reservoirs contain gas in free form (gaseous). Due to its low density it will be found at the top of the structure given enough time for gravity-capillary equilibrium to take place (see two phase module on equilibrium). This low density also implies greater overpressure at depth as compared to similar structures containing oil only (lecture Reservoir Engineering). Figure 1 illustrates a conceptualized saturated reservoir. In the mathematical formulation of such a problem, the gas phase is considered to have a free and a dissolved component, whereas the oil phase is considered stagnant.

![Figure 1: Typical three phase (saturated) reservoir featuring gas, oil and water zones and contacts. Of course, gravity-capillary driven transition zones apply as presented in two phase flow. In the numerical formulation of such a problem, the gas phase is considered to change from gaseous to liquid (and vice versa) depending on PVT conditions.](image)

As presented in two-phase flow, the governing equations are derived from a conservation law and Darcy’s relationship between pressure and flow. This translates directly to the oil and water phase. For the gas phase, however, an additional component is introduced, allowing distinguishing between gas in liquid and gaseous state. The concept of relative permeabilities remains as measure of relative flow remains.

Pressure and Fluid Properties

In the black-oil formulation temperature is constrained and not solved for. Fluid properties such as solution gas-oil ratio (Rs), oil/gas formation volume factors (Bo/Bg), phase viscosities ($\mu_o/\mu_w$) are computed for a given pressure range but at constant temperature. This is illustrated in Figure 2, where the shaded zone reflects possible states in the reservoir of different pressures. These (local) differences in pressure may result from injection/production or depletion for example.
Figure 2: Pressure-Temperature diagram for a hydrocarbon accumulation showing the corresponding composition w.r.t. gas and oil saturation.

Look-up tables for properties are created based on PVT correlations upfront the simulation. At any point in the reservoir, the simulator uses those to assess flow properties depending on the local (grid cell) pressure. We’ll have a detailed look at the generated tables in class when setting up the components/fluid models.

Figure 3: Example of pressure dependent flow properties: solution factor and formation volume factor. These will vary over the simulation domain as pressure changes locally.
The Formulation

For the sake of simplicity we look at the governing equations in reservoir volume terms only, meaning that the formation volume factor for the three acting phases is neglected. The equations defining transient fluid flow for the water and oil phase are straightforward:

\[
\phi \frac{\partial S_w}{\partial t} - \nabla \cdot q_w + q = 0
\]  \hspace{1cm} (1)

For the water phase and

\[
\phi \frac{\partial S_o}{\partial t} - \nabla \cdot q_o + q = 0
\]  \hspace{1cm} (2)

for the oil phase. We already mentioned that the gas phase is

\[
\phi \frac{\partial (S_g + R_{so}S_o)}{\partial t} - \nabla \cdot \left( q_g + R_{so}q_o \right) + q = 0
\]  \hspace{1cm} (3)

The flow rate in eqts. (1) to (3) for water, oil and gas \((q_w, q_o \text{ and } q_g \text{ respectively})\) are given by the very familiar multiphase version of Darcy’s law of filtration:

\[
q_i = -k \frac{k_{ri}}{\mu_i} \nabla (p_i - \rho_i g z)
\]  \hspace{1cm} (4)

Which yields for equations (1) to (3)

\[
\phi \frac{\partial S_w}{\partial t} - \nabla \cdot \left( -\frac{k k_{rw}(S_w)}{\mu_w} \nabla (p_w - \rho_w g z) \right) + q_w = 0
\]  \hspace{1cm} (5)

\[
\phi \frac{\partial S_o}{\partial t} - \nabla \cdot \left( -\frac{k k_{ro}(S_o)}{\mu_o} \nabla (p_o - \rho_o g z) \right) + q_o = 0
\]  \hspace{1cm} (6)

\[
\phi \frac{\partial (S_g + R_{so}S_o)}{\partial t} - \nabla \cdot \left( -\frac{R_s k k_{ro}(S_o)}{\mu_o} \nabla (p_o - \rho_o g z) + \frac{k k_{rg}(S_g)}{\mu_g} \nabla (p_g - \rho_g g z) \right) + q_g = 0
\]  \hspace{1cm} (7)

accounting for formation volume factors

\[
\phi \frac{\partial (S_w)}{\partial t} - \nabla \cdot \left( -\frac{k k_{rw}(S_w)}{B_w \mu_w} \nabla (p_w - \rho_w g z) \right) + q_w = 0
\]  \hspace{1cm} (8)

\[
\phi \frac{\partial (S_o)}{\partial t} - \nabla \cdot \left( -\frac{k k_{ro}(S_o)}{B_o \mu_o} \nabla (p_o - \rho_o g z) \right) + q_o = 0 \hspace{1cm} (9)
\]
assuming pressure independent (constant) porosity. Viscosities, densities, solution factors are evaluated depending on local pressure. Where the sum of phase saturation always yields unity (constraint equation).

\[
\sum_{i=1}^{3} S_i = 1 \quad (11)
\]
Three Phase Relative Permeabilities

The concept of relative permeabilities as factor of how much of the single phase permeability is available to either phase flow still applies. Graphically, however, one uses triangle diagrams to display the dependence of $k_r$ on phase saturation.

![Triangle Diagram](image)

**Figure 4:** Three phase relative permeabilities as function of saturation – here: Kro (oil relative permeability).

Figure 5 illustrates that at each corner the saturation of the respective phase is one and is decreasing linearly with moving a line parallel to the opposing side away from the phase saturation endpoint (corner-points).
Figure 5: Conceptualization of three phase relative permeabilities. Once saturation of two phases is known, the third follows from $Sw+So+Sg=1$. The intersection point provides the relative permeabilities of each phase, generated from interpolation between the isolines.

Along the bounding edges of the triangle, relative permeabilities equal to those of a two-phase system, since the saturation of the opposing edge phase is zero.
Finite-Difference Grids and Initialization

Commercial black-oil reservoir simulators (Schlumberger’s E100, CMG’s Imex, Roxar’s Tempest...) use regular grid as means of space discretization which allows for a finite volume discretization of the governing partial differential equations (i.e. (8) to (10)). We’ll cover lots of details during class with the use of the CMG manuals, a brief introduction on finite difference simulators is provided here.

The Grid

All cells are represented as hexahedra, that are cubes with 6 faces and 8 corner nodes. The grid is constructed either using a block centered or a corner point strategy: in the first case a point cloud represents cell barycenters around which the grid blocks are formed, the latter indicates that the corner nodes are provided in a coordinate system.

![Figure 6: A regular grid discretization of a reservoir. As covered in the previous module, each entity (cell, grid block) has uniform properties (here: permeability in x/y direction)](image)

The regular nature of such grids allows each grid cell to be addressed using three integers, namely i, j and k. These represent axis in x, y and z direction respectively. So a grid block (0,10,0) is to be found at the upper most layer (k equals zero) in the first row along the x direction and 10 cells deep in the y direction. Computationally, this convention allows for fast access of variables associated with cells in terms of arrays.
Initialization

Provided a pressure at a certain reference depth, vertical location of the oil-water contact and capillary pressure curves, a initialization computation is conducted to arrive at capillary pressure-saturation equilibrium as outlined in the two-phase module. Based on capillary pressure curves (rock property) and the densities of the present phases, a force balance between oil-phase pressure (buoyancy) for each cell above the oil-water contact is established, leading to initial values for fluid pressure (honoring the reference pressure) and saturation.
Non-Linearity

Since for the advection equation

$$\phi \frac{\partial S_w}{\partial t} \nabla \cdot q_w + q_w = 0$$  \hspace{1cm} (12)

the phase flow rate $q_i$ depends on the phase relative permeability which itself is a function of saturation

$$q_i = k \frac{k_{ri}(S_i)}{\mu_i} \nabla p$$  \hspace{1cm} (13)

equation (12), if solved for saturation (i.e. how much of oil, water and gas is where and when), the solution of the variable (here $S_w$) depends on itself, which introduces non-linearity and requires for according solution techniques.

$$\phi \frac{\partial S_w}{\partial t} - \nabla \cdot \left( k \frac{k_{rw}(S_w)}{\mu_w} \nabla (p_w - \rho_w g z) \right) + q_w = 0$$  \hspace{1cm} (14)

Figure 9 illustrates the dependence flow properties such as relative permeability and viscosity on saturation and pressure and vice versa.

![Diagram](image)

Figure 9: Solving for saturation, pressure and flow properties iteratively as a mean to handle non-linearity.

Usually, non-linear systems are solved for iteratively until convergence is achieved. During these iterations, values for $R_s$ and $\mu$ are obtained in dependence on pressure and temperature (which are changing during the solution stage) from so called look-up tables which base on (empiric) correlations.
Figure 10: A PVT lookup table. This is used to iteratively arrive at a solution for pressure and saturation.

Due to the large change in volume, and hence in pressure, this convergence is critical near bubble point conditions, that is where a system goes from having no gas to a system that has free gas or the other way round.

![Diagram]( undersaturated state $p > p_b$)

$S_i, p$

![Diagram]( saturated state $p < p_b$)

$S_i, p$

Figure 11: Near bubble point pressure, iterative solution of saturation and pressure will eventually converge towards either side.
Related BSc Examination Concepts

- Relative Permeability
- Phase Diagrams
- Dew Point Curve
- Bubble Point Curve
- Lookup tables for simulators
- Governing equations / conservation laws: two-phase incompressible versus Black Oil model
References


